Modeling Tokamak Transients with M3D-C1

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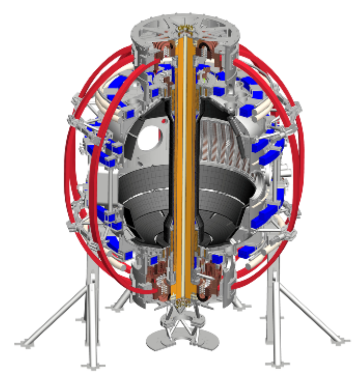
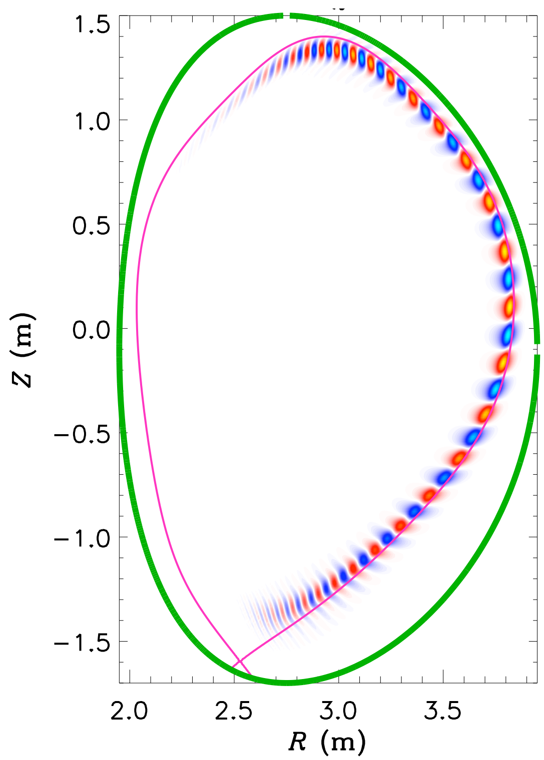


"Transients" Identified as Major Challenge to Successful Tokamak Reactor

- Edge Localized Modes (ELMs)
  - Intermittent bursts of heat from plasma edge
  - Present in most H-mode scenarios
  - Understood to be ideal-MHD instabilities of the plasma edge (peeling-ballooning modes)
  - Expected to melt / erode divertor in ITER if not mitigated
- Disruptions
  - Rapid, uncontrolled loss of plasma current and thermal energy
  - Cause significant heat loads on walls and forces on conducting structures
  - Can cause relativistic electron beams (runaways)

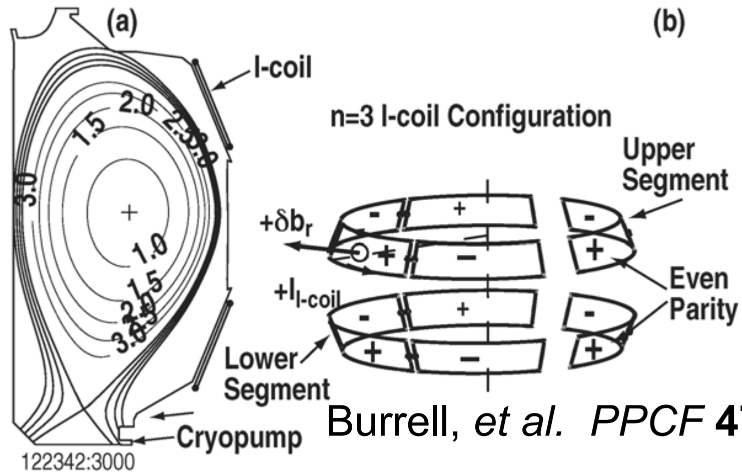


- "ITER and later reactors will require very large reductions in the magnitude and frequency of both ELMs and major disruptions based on extrapolations from current experiments"

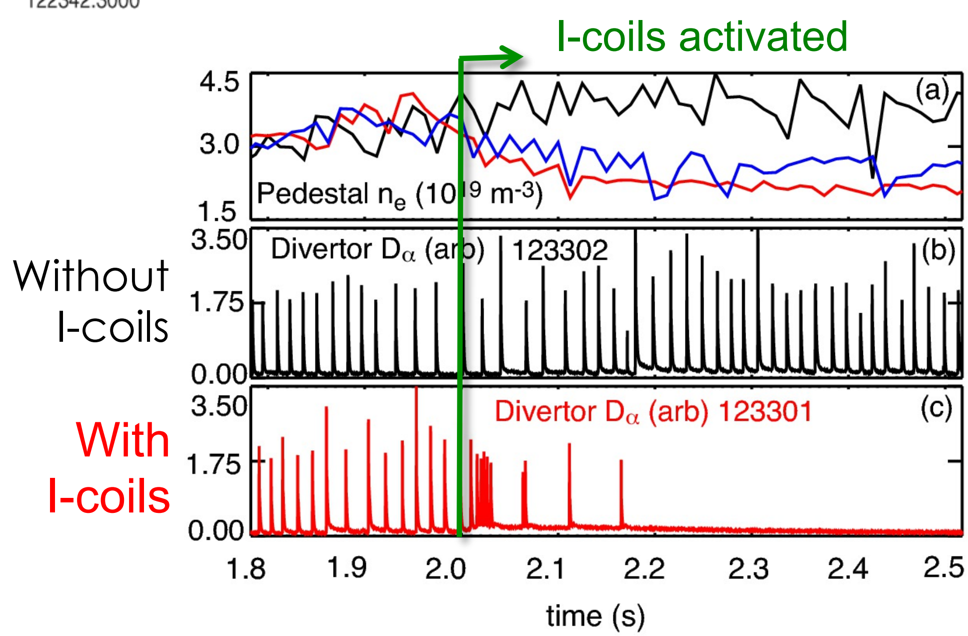
http://science.energy.gov/~/media/fes/pdf/program-news/Transients_Report.pdf

RMPs are a Primary Strategy for ELM Mitigation

- ELMs can be completely suppressed by applying non-axisymmetric Resonant Magnetic Perturbations (RMPs)
- Works on many tokamaks
  - DIII-D, AUG, KSTAR
- Doesn't work on others
  - NSTX, MAST, JET
- Only works for certain conditions
  - q_{95} windows, collisionality/density thresholds
- **We can't predict when RMP ELM suppression will work**
  - **This presents big risks for ITER!**



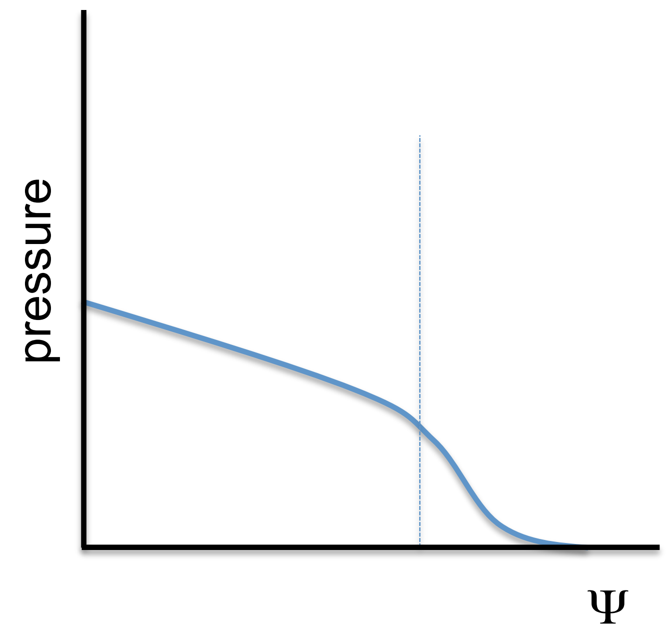
Burrell, *et al.* *PPCF* **47** (2005) B37



Evans, *et al.* *Phys. Plasmas* **13** (2006)

EPED Model Suggests Suppression Due to Enhanced Transport at Pedestal Top

- EPED Model of pedestal structure:
  - Gradient determined by local KBM stability
  - Width grows until global P-B stability threshold is reached (ELM)



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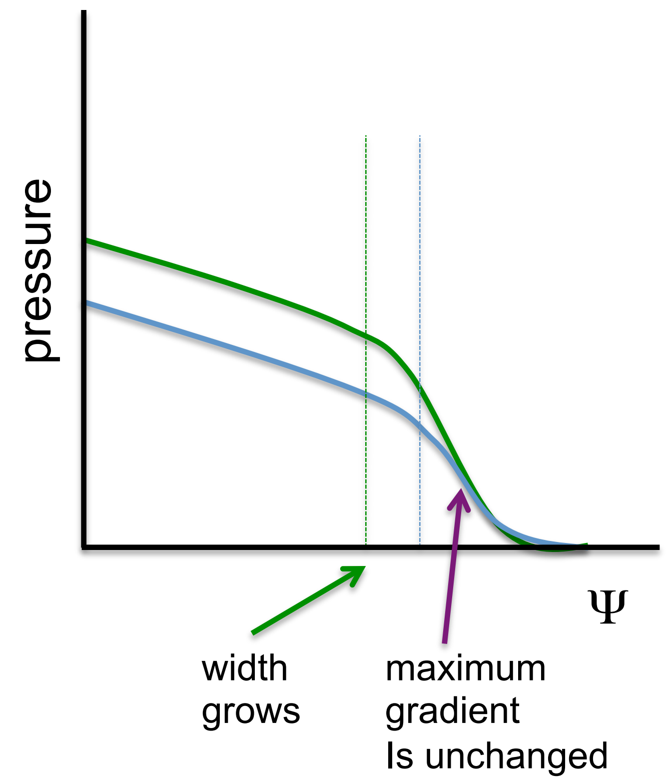

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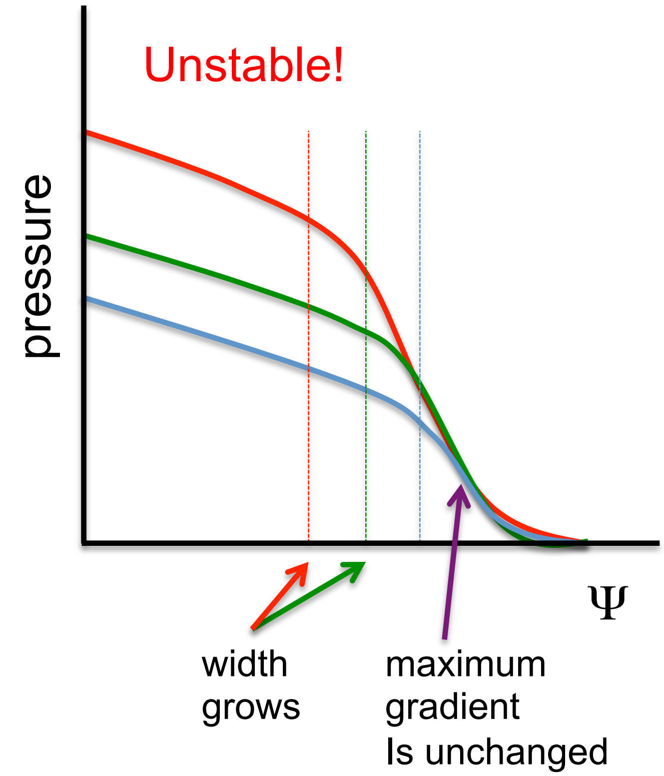

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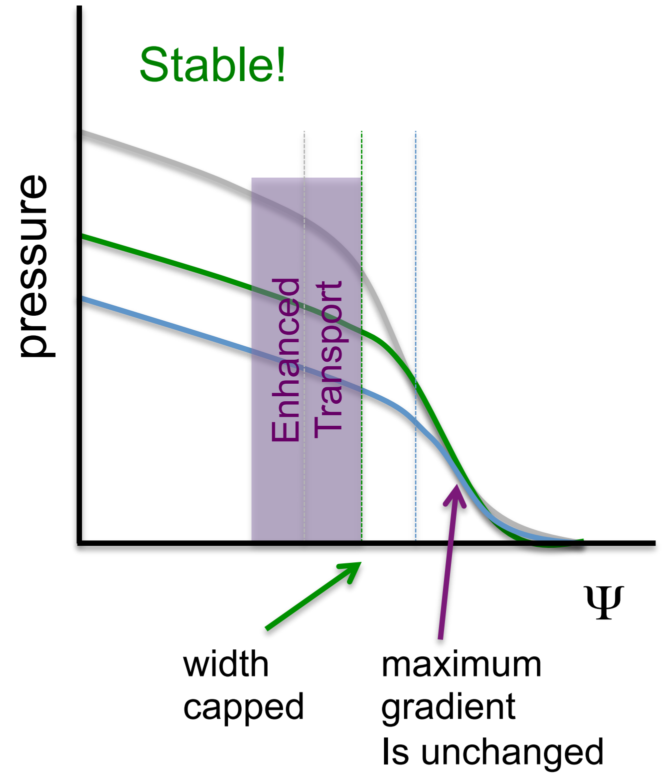
  - Something stops widening of pedestal before threshold
  - Requires enhanced transport at $\Psi \approx 96$–97%



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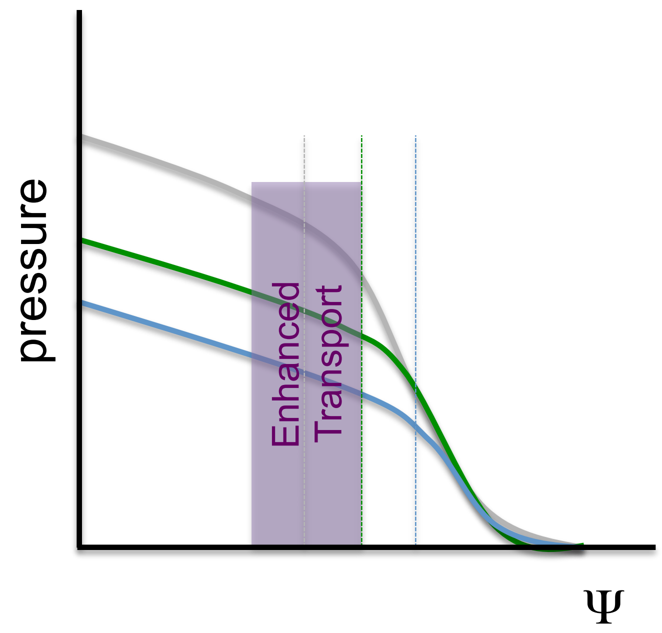
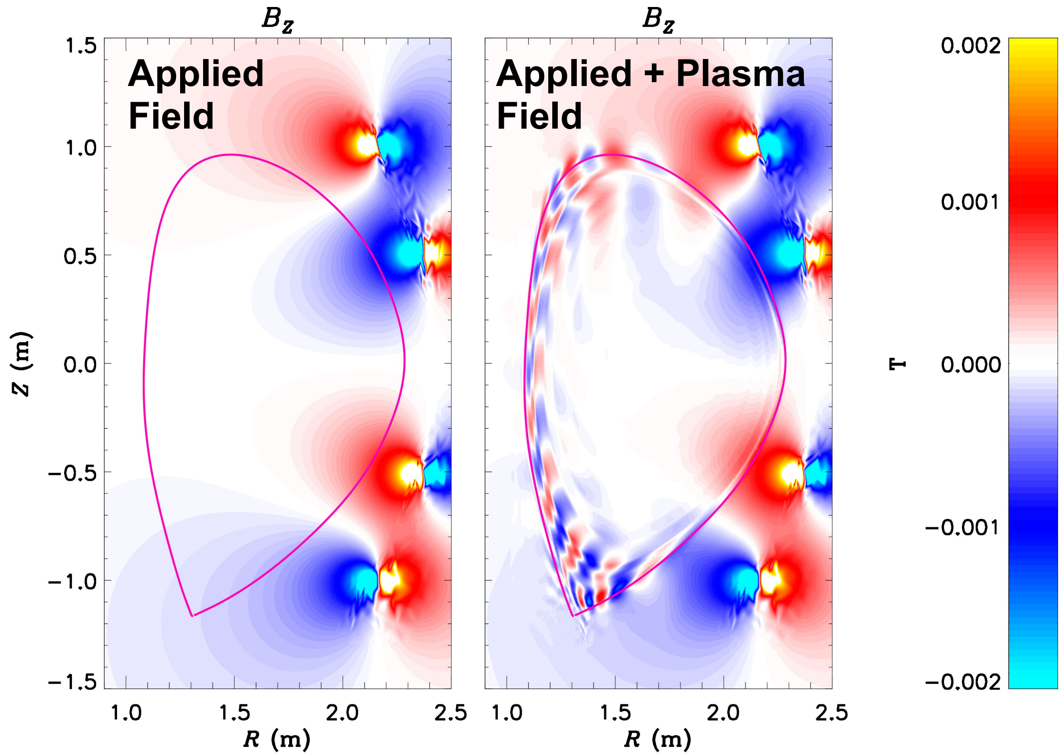


- Implies model of ELM suppression:
  - Something stops widening of pedestal before threshold
  - Requires enhanced transport at $\Psi \approx 96$–97%
- Predictive modeling needs model of RMP effect on transport
  - Enhanced neoclassical transport? Turbulent transport (KBM)? Stochasticity → parallel transport?
- Answering these questions requires knowing 3D equilibrium

MHD Response Plays Major Role in 3D Tokamak Equilibrium

- Perturbing field causes equilibrium to be non-axisymmetric
- Non-axisymmetric response currents in the plasma are a major contribution to perturbed equilibrium
  - Perturbed equilibrium is generally very different from axisymmetric equilibrium + applied 3D fields



- Need MHD codes to calculate perturbed equilibrium
  - IPEC (linear, ideal)
  - MARS (linear, single-fluid resistive)
  - **M3D-C1 (linear/nonlinear, two-fluid resistive)**

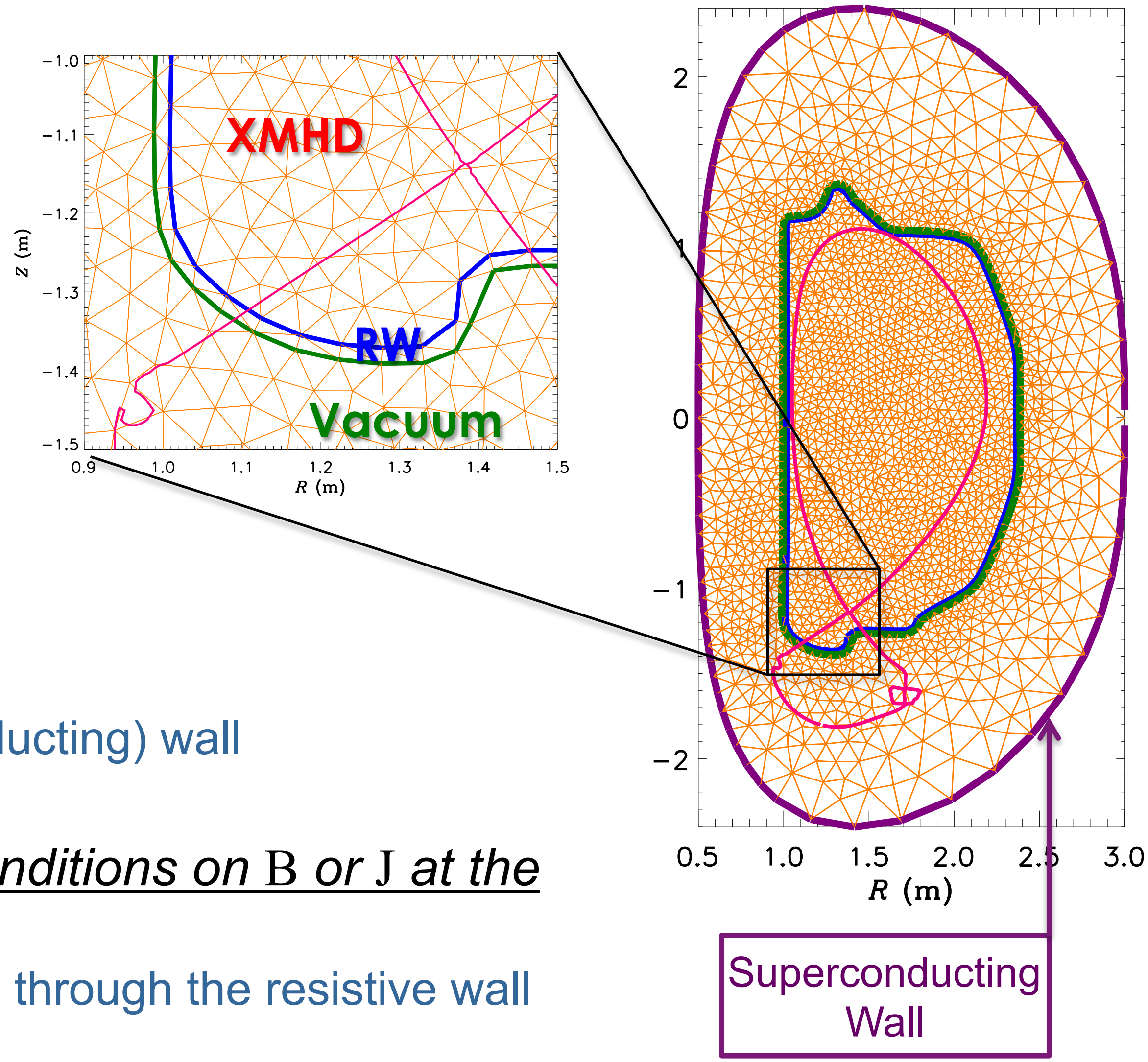
M3D-C1 Is Parallel, Finite-Element Code Using Unstructured, Multi-Region Mesh

- Triangular C1 finite elements on unstructured mesh

- 3 regions inside domain:
  - XMHD (Extended MHD)
  - RW ($\mathbf{E} = \eta_W \mathbf{J}$)
  - Vacuum ($\mathbf{J} = 0$)

- Boundary conditions:
  - $\mathbf{v}$, p, n set at inner wall
  - $\mathbf{B}$ set at outer (superconducting) wall

- <u>*There are no boundary conditions on* B *or* J *at the resistive wall*</u>
  - Current can flow into and through the resistive wall



Two-Fluid Extended MHD Model

$$\frac{\partial n}{\partial t} + \nabla \cdot (n_i \mathbf{v}) = 0$$

$$n_i m_i \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = \mathbf{J} \times \mathbf{B} - \nabla p - \nabla \cdot \Pi_i$$

$$\frac{\partial p}{\partial t} + \mathbf{v} \cdot \nabla p + \Gamma p \nabla \cdot \mathbf{v} = -\frac{1}{n_e e} \mathbf{J} \cdot \left(\Gamma p_e \frac{\nabla n_e}{n_e} - \nabla p_e \right) - (\Gamma - 1) \nabla \cdot \mathbf{q}$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

$$\mathbf{E} = -\mathbf{v} \times \mathbf{B} + \eta \mathbf{J} + \frac{1}{n_e e} (\mathbf{J} \times \mathbf{B} - \nabla p_e)$$

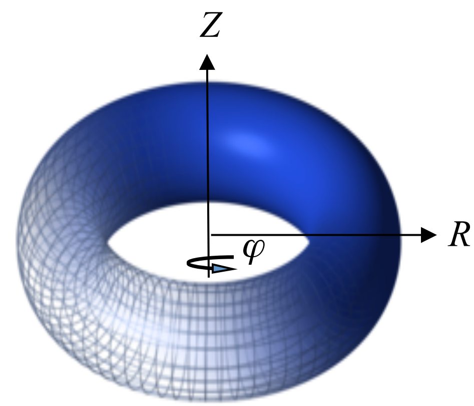
$$\Pi_i = -\mu \left[\nabla \mathbf{v} + (\nabla \mathbf{v})^T \right] + \Pi_i^{gv} + \Pi_i^{\parallel}$$

$$\mathbf{q} = -\kappa \nabla T_i - \kappa_{\parallel} \mathbf{bb} \cdot \nabla T_e$$

$$\mathbf{J} = \nabla \times \mathbf{B}$$

$$\Gamma = 5/3$$

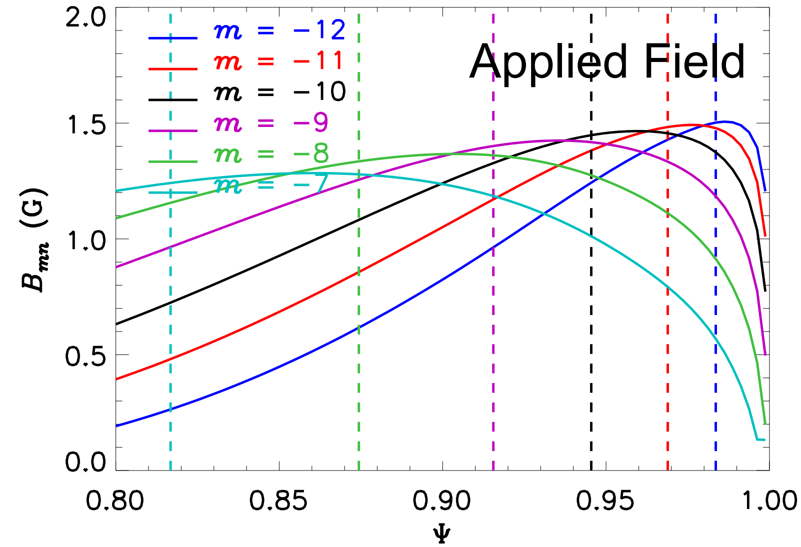
$$n_e = Z_i n_i$$

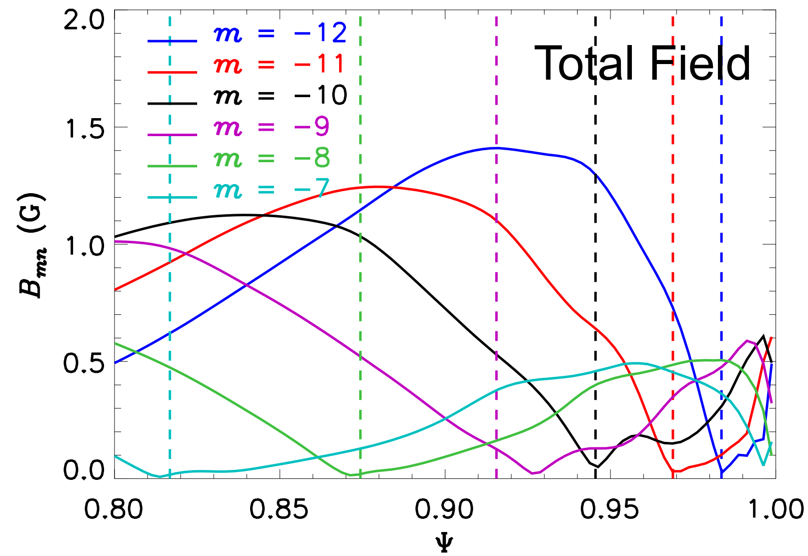


- (R, φ, Z) coordinates → no coordinate singularities in plasma
- Boundary conditions:
  - Linear, time-independent (**plasma response**) – single n
  - Linear, time-dependent (**linear stability**) – single n
  - Nonlinear, time-dependent (**nonlinear evolution**) – toroidal finite elements

Linear MHD Modeling Shows "Kinking," "Screening," and "Tearing" in Response

- **Kinking**: amplification of non-resonant field components
  - Makes distortion of surfaces larger than implied by applied fields
- **Screening**: reduction of resonant field components
  - Makes islands smaller than implied by applied fields
- **Tearing**: when plasma response fails to screen resonant components
  - Only possible in non-ideal response





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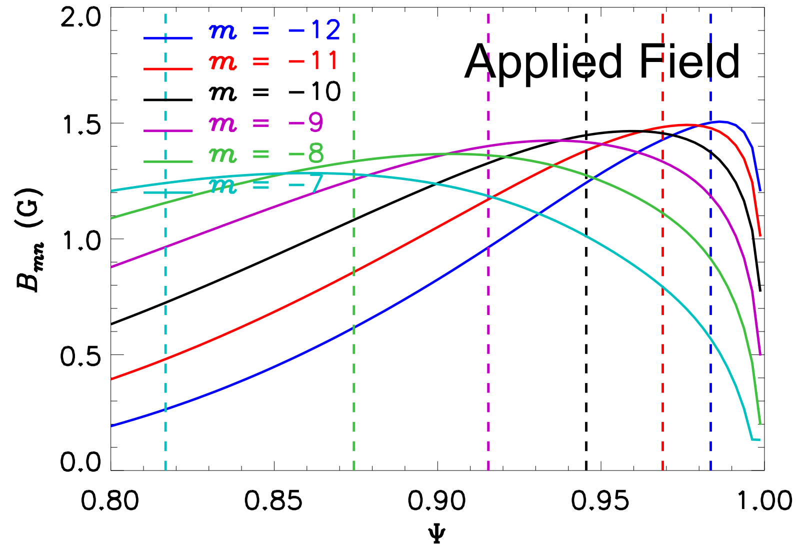

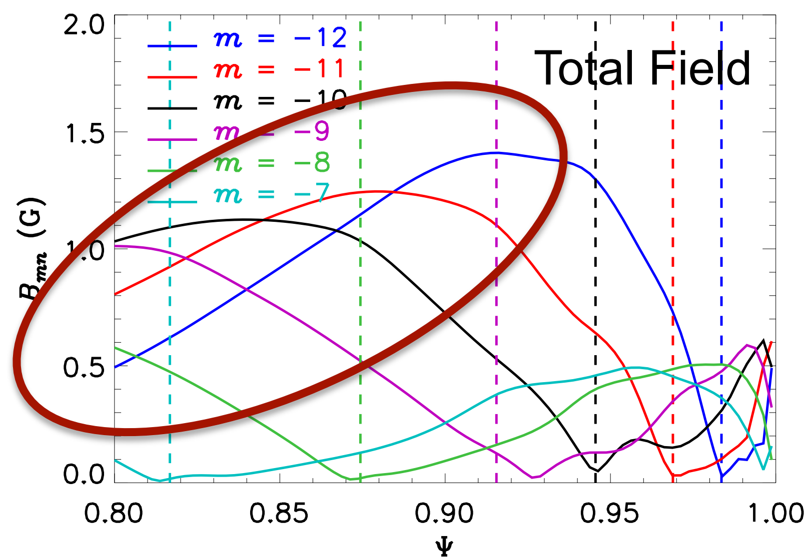

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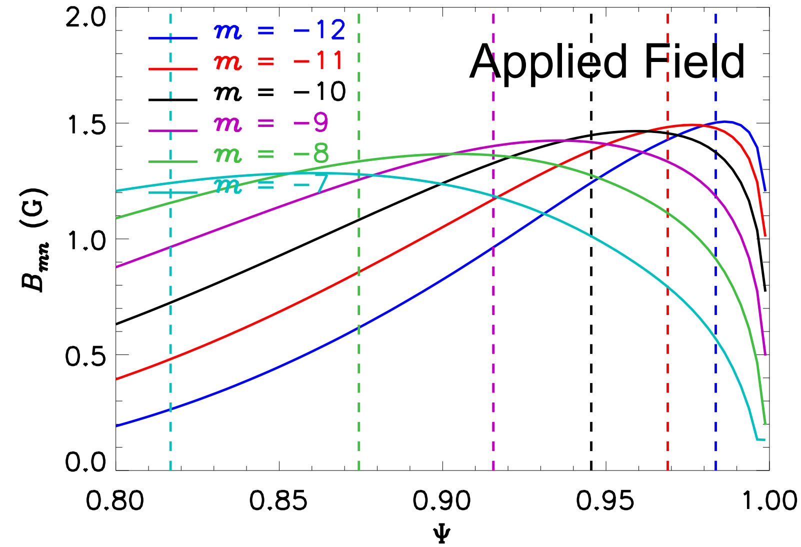

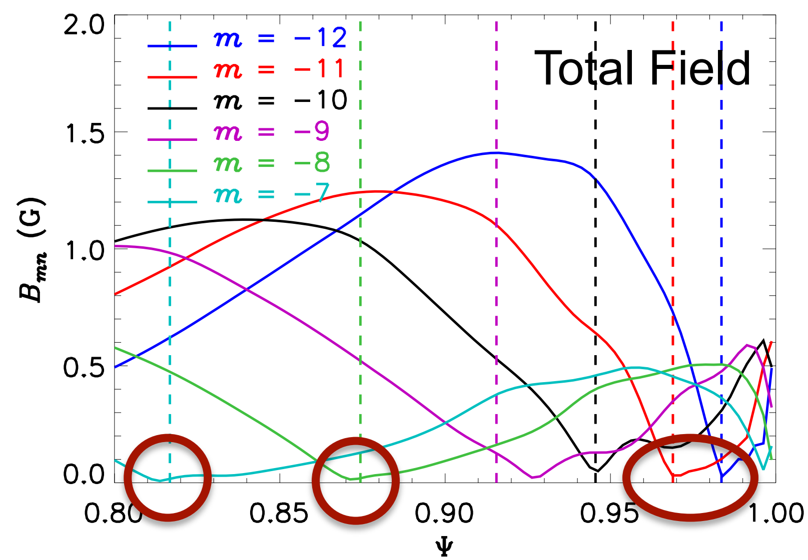

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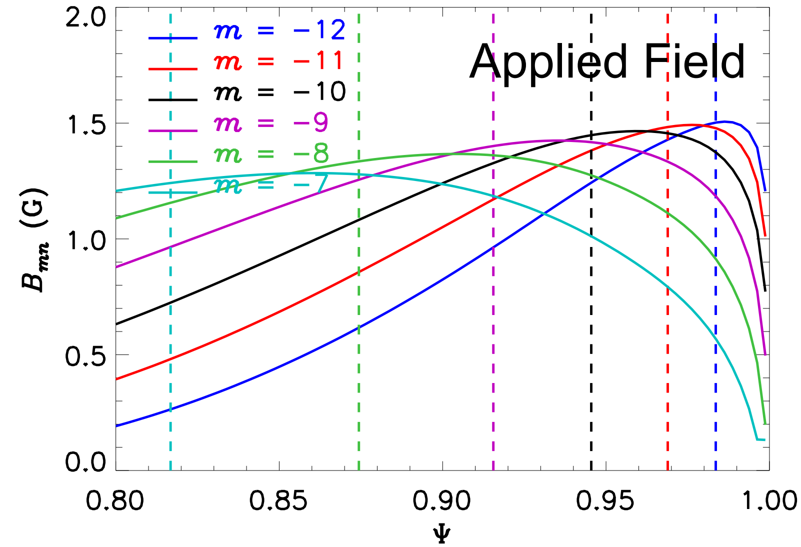
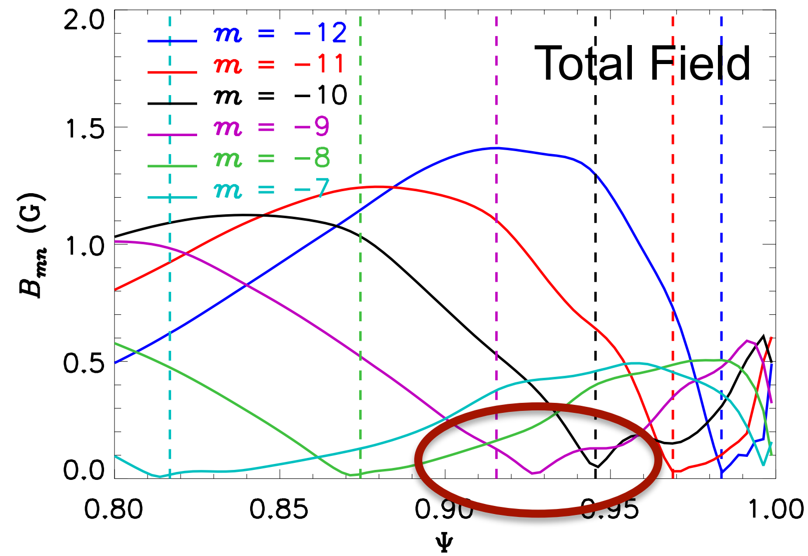
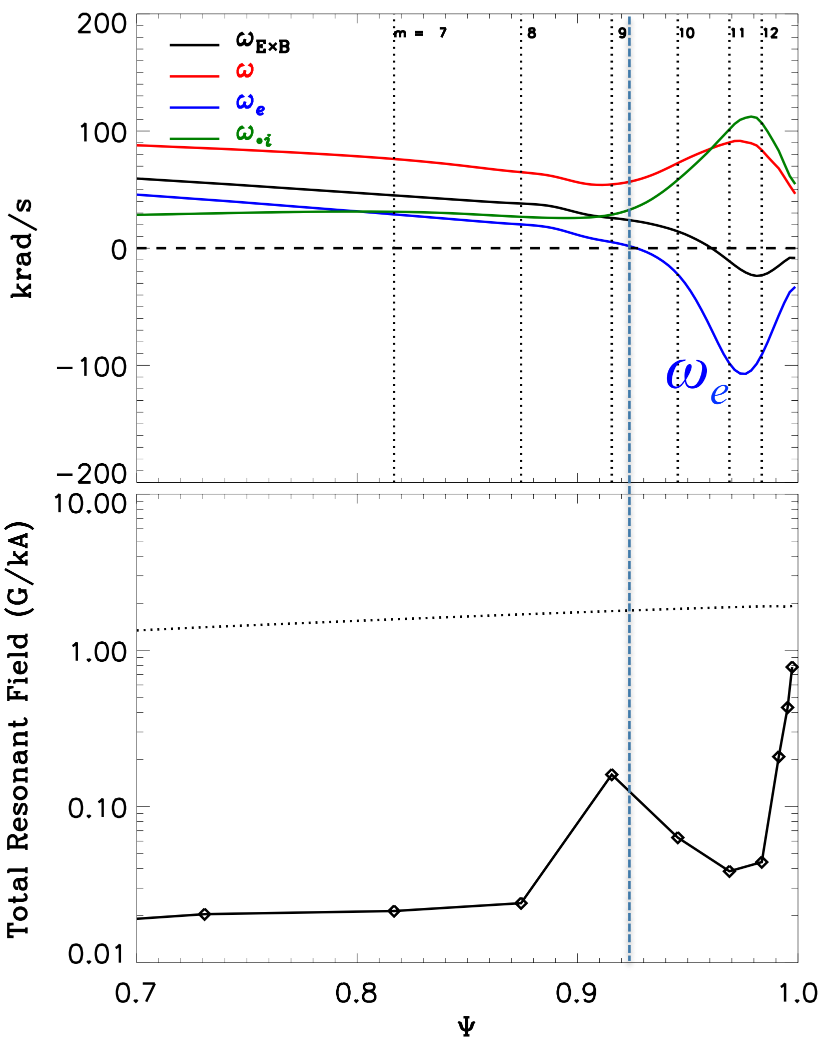
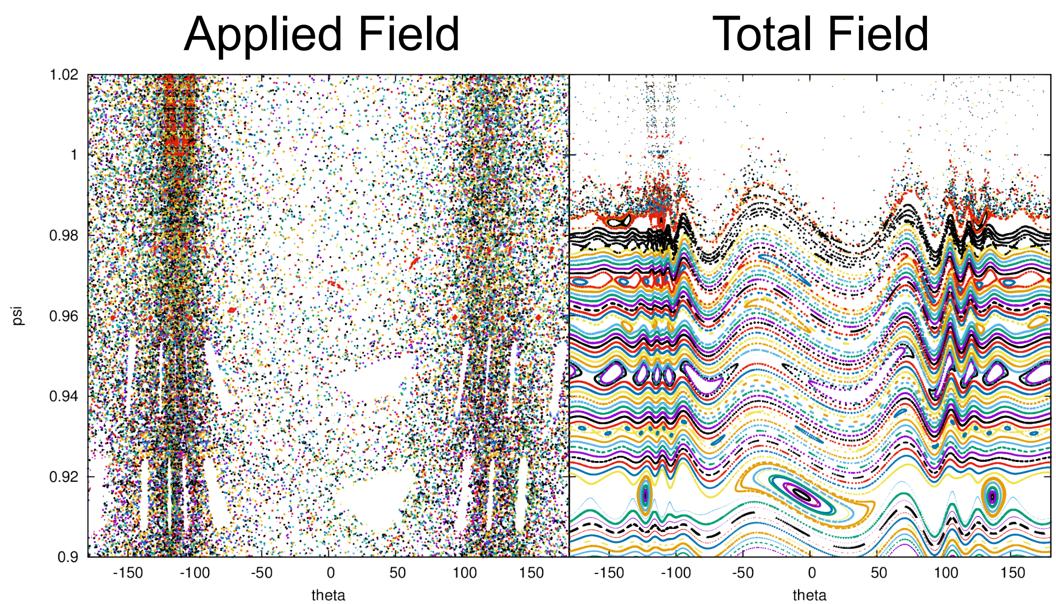




Tearing Response is Greatest Where Electron Rotation is Small





- Plasma Response mostly screens islands

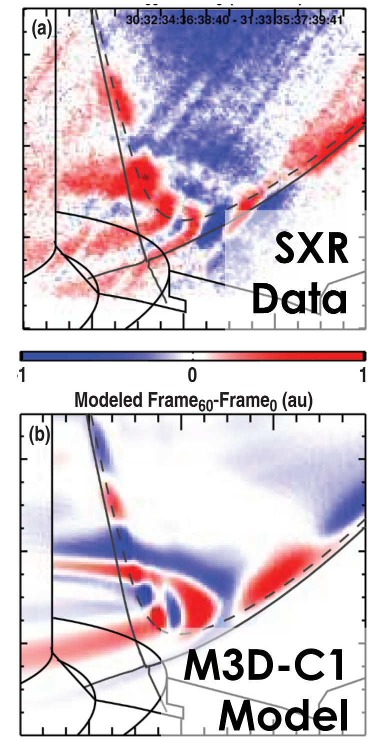
- Tearing occurs where ω_e is small

$$\omega_e = \omega_{E\times B} + \omega_{e*} \qquad \omega_i = \omega_{E\times B} + \omega_{i*}$$

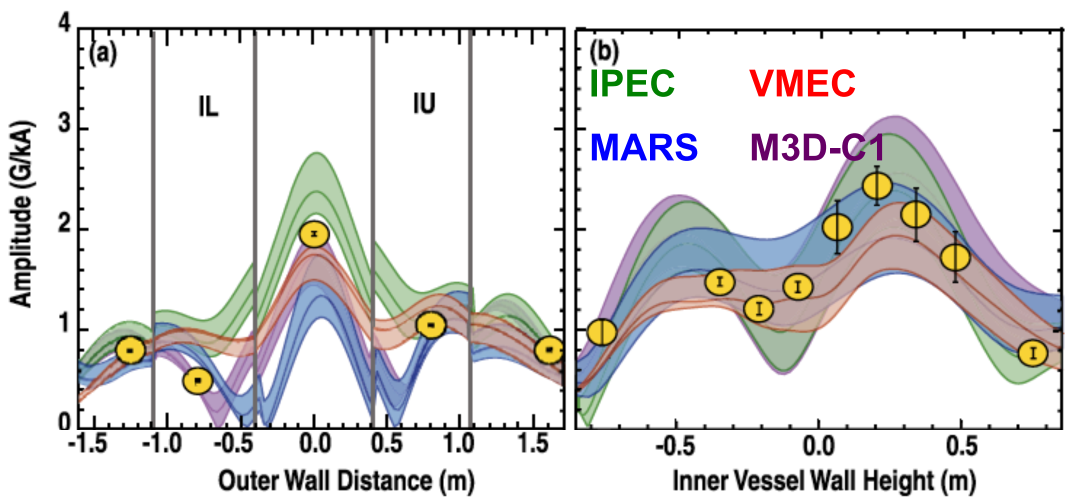
$$\omega_{e*} = -p_e'(\psi)/n_e e \qquad \omega_{i*} = p_i'(\psi)/Z_i n_e e$$

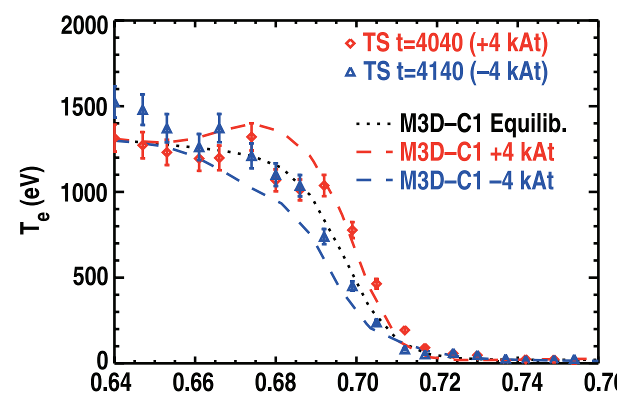
Experiments Clearly See "Kink" Response

- Including plasma response is necessary to accurately model edge measurements
  - T_e, n_e profiles in edge strongly affected by "kink" response
  - Linear modeling is successful in reproducing measured profiles; magnetics data



NM Ferraro, *et al.* *Nucl. Fusion* **53**, 073042 (2013)

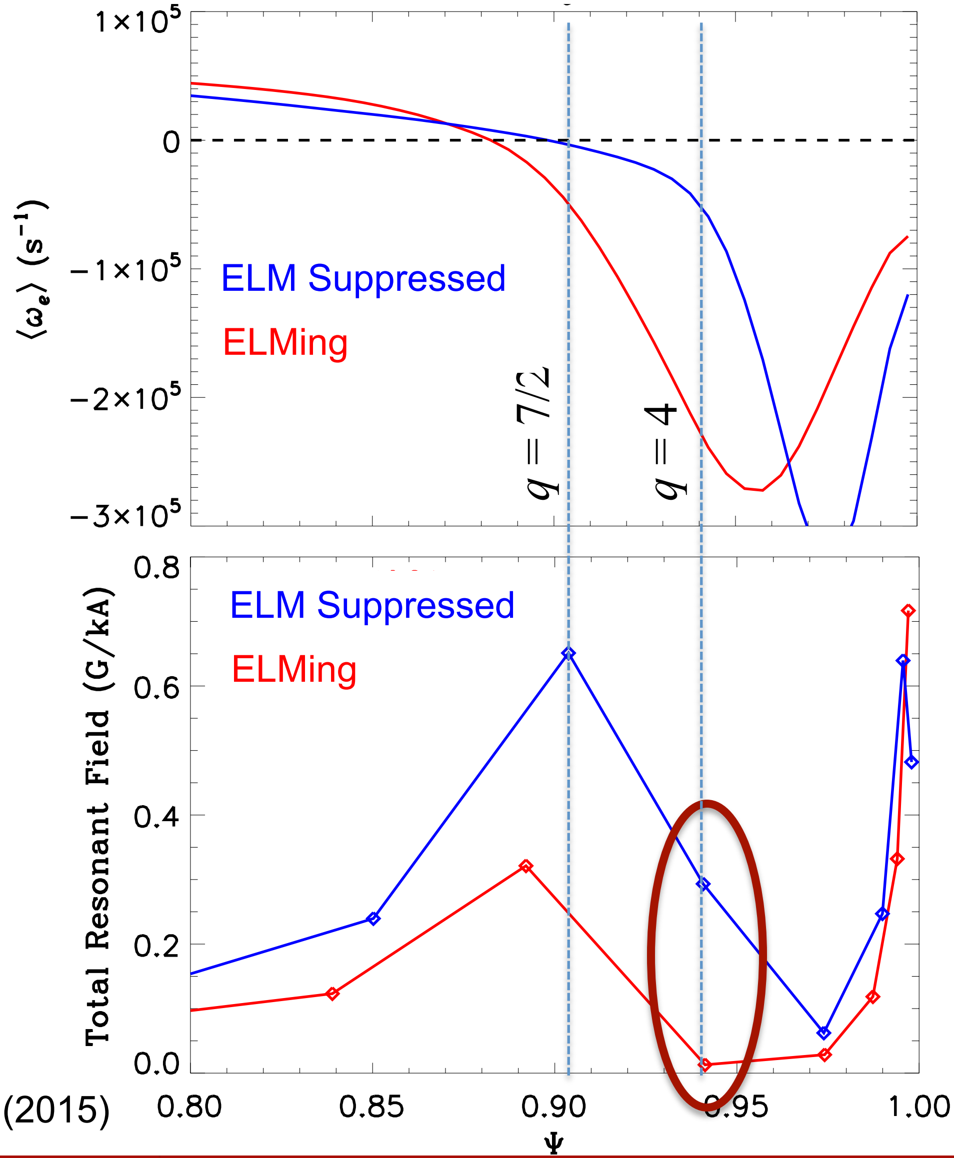




JD King, *et al.* *Phys. Plasmas* **22**, 072501 (2015)

Significant Enhancement of Tearing Response Found in ELM-Suppressed State

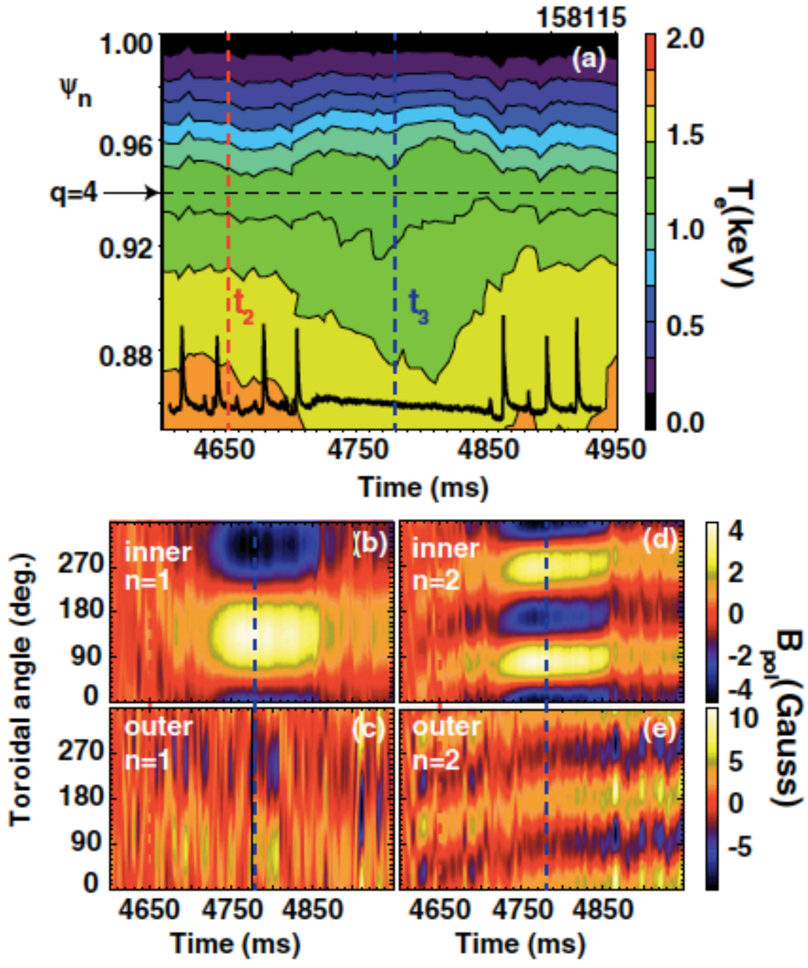
- Experiment applied $n = 2$ fields with rotating phasing
  - Phase of upper coil held constant, phase of lower coil rotated
- Plasma enters ELM-suppressed state near “even parity” phasing
- Measurements show change of rotation and pressure profiles in ELM-suppressed state
- Modeling shows enhanced tearing near pedestal top in ELM-suppressed state
  - $\omega_e = 0$ moves very close to $q = 7/2$ surface



Nazikian, *et al.* *PRL* **114**, 105002 (2015)

Experiments See Hints of Island Formation

- Measuring small islands (~1 cm) is very difficult experimentally
- In transition into ELM-suppressed state, a bifurcation similar to the formation of a locked island is observed
  - Temperature flattening near top of pedestal
  - Non-rotating magnetic signal
- No island is seen directly. Modeling is still needed to understand results
  - Truly predicting island formation requires nonlinear modeling



Nazikian, *et al.* *PRL* **114**, 105002 (2015)

Summary of RMP ELM Suppression Modeling

- We think we have a good understanding of the perturbed equilibrium
  - Plasma tends to eliminate stochasticity, except possibly near ω_e=0 location
  - Kinking response plays big role in geometry; validation bears this out
- We don't yet have a good understanding of transport in the perturbed equilibrium
  - EPED model could explain RMP ELM suppression with enhancement of transport near Ψ ~ 96%
  - Need to couple "transport" codes to 3D equilibrium!
    - In progress with NEO3D, GTC, SPIRAL, XGC
- Tearing at top of pedestal in two-fluid modeling is suggestive
  - Could an island be the source of the transport?
  - This is very difficult to measure experimentally
- We've gotten far with linear modeling, but nonlinear modeling may be required
  - Experimental evidence of bifurcation into ELM suppressed state
  - Opening of island > 1 cm is a nonlinear process

Disruptions

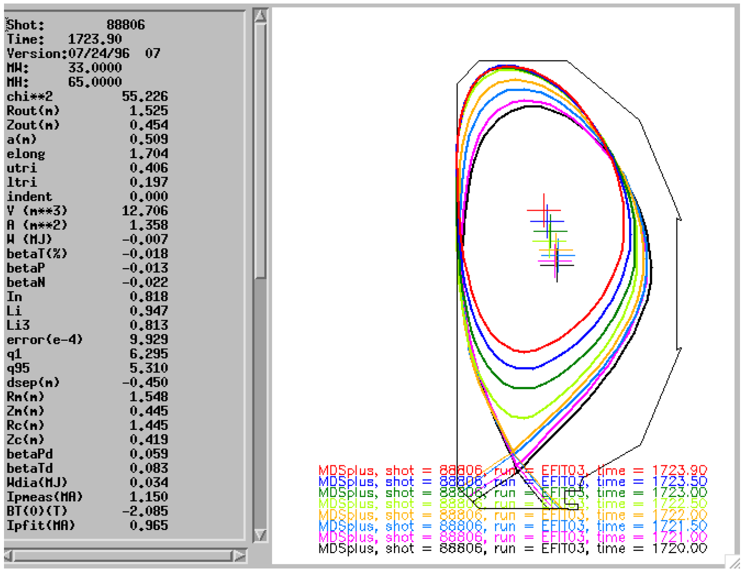
- There are many causes of disruptions in tokamaks
  - Locked tearing modes; External kinks / Resistive Wall Modes (RWMs); Density (Greenwald) limit; Radiation collapse

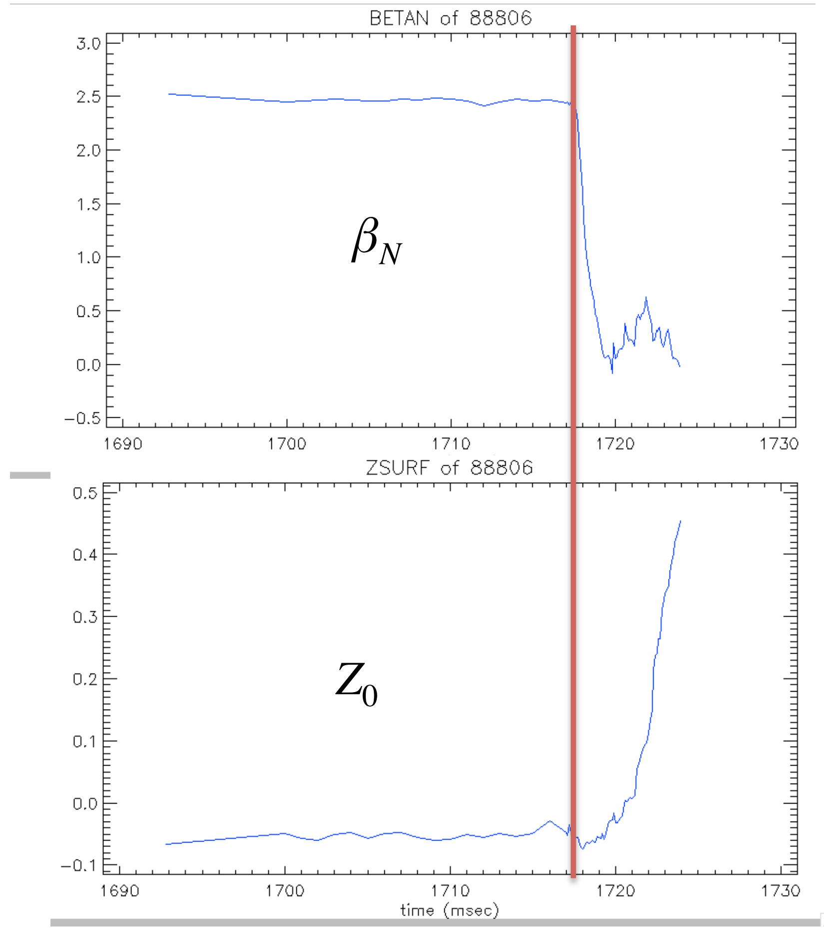
- Disruptions have two main components:
  - **Thermal Quench (TQ)**: loss of thermal energy
    - May be due to plasma hitting wall or radiation
  - **Current Quench (CQ)**: dissipation of plasma current
    - Generally involves loss of control; plasma hits wall
    - Large currents are induced in wall (eddy currents) and flow from plasma to wall (halo currents)

- Our focus is on understanding CQ phase
  - M3D-C1's resistive wall model gives unique tool for modeling CQ

- ITER's concerns for CQ phase:
  - Generation of runaway electrons
  - Forces on conducting structures (where and how much). Non-axisymmetric forces are especially problematic.

Vertical Displacement Events Result From Loss of Vertical Stability Control





- VDEs may be caused by thermal quench (cold VDE), or may cause thermal quench (hot VDE)

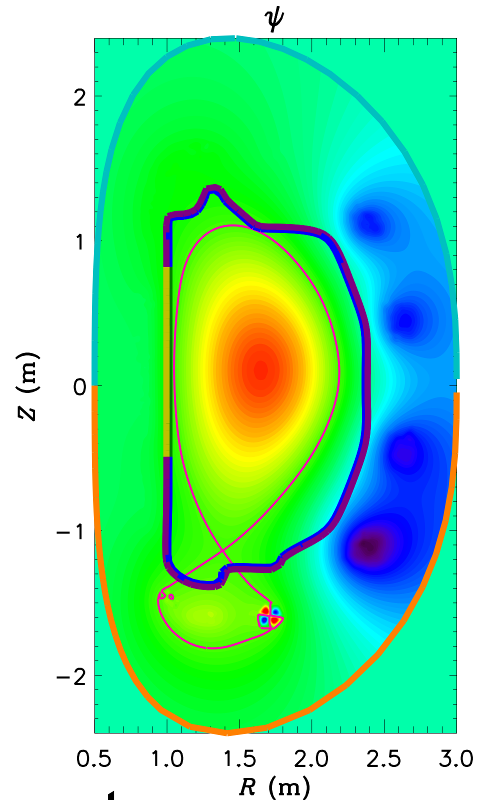
- We are interested in how current quench evolves in both cases
  - Eddy currents; halo currents; non-axisymmetries

Disruption Simulations Initialized using Vertically Unstable EFIT Reconstructions

- Nonlinear calculations use fairly realistic plasma parameters
  - Spitzer resistivity: $S_0 \approx 6.8\times10^7$
  - Anisotropic thermal conductivity:

$$\chi_{\parallel}/\chi_{\perp} = 10^6$$

- RW region approximates first wall, not vacuum vessel here



- Cold-VDE calculations have anomalous χ to cause TQ before vertical instability
- Hot-VDE calculations have lower χ and remain hot until after plasma touches wall

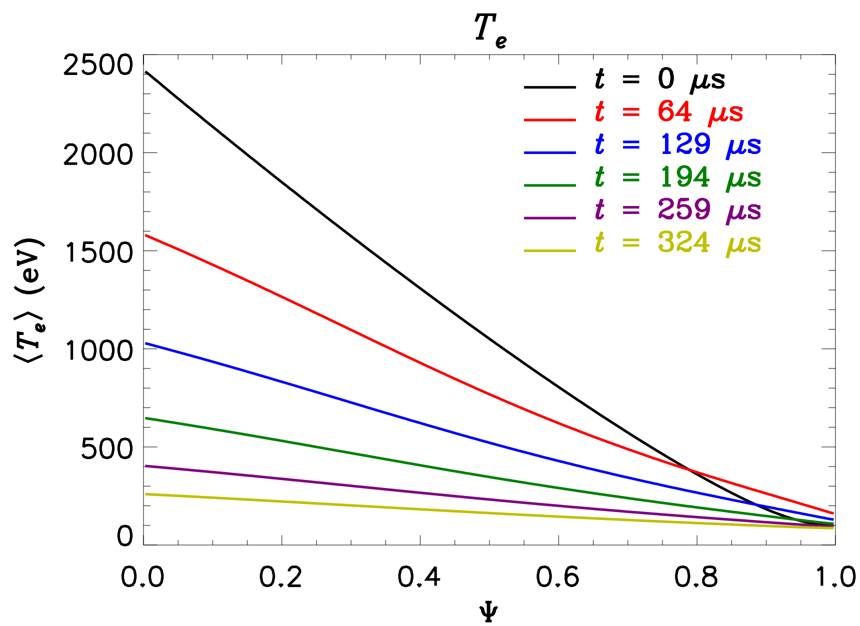
“Cold-VDE” Features Thermal Quench Before Vertical Instability

- Thermal Quench (TQ) is modeled by including anomalous thermal conductivity

$$100 < \chi_\perp < 800\ \mathrm{m}^2/\mathrm{s}$$

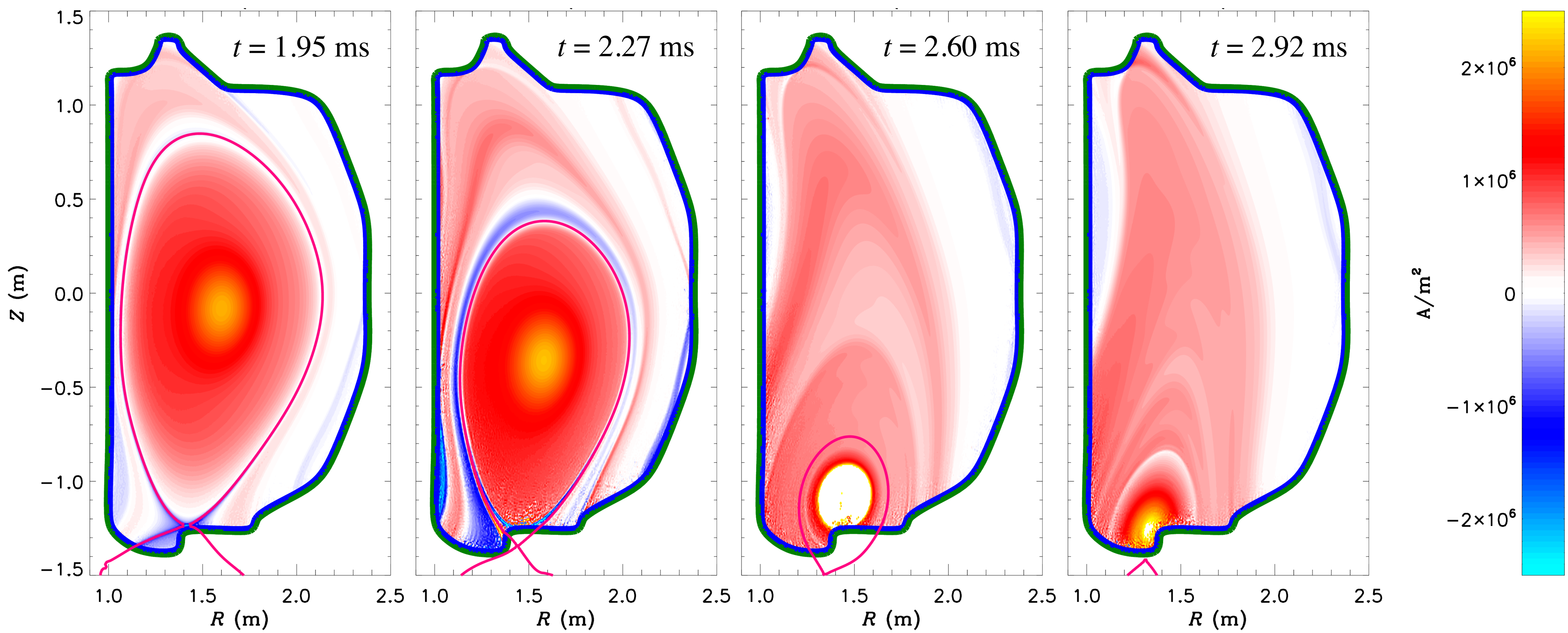
- Thermal quench happens on $\sim 100\ \mu\mathrm{s}$ timescale

- (TQ phase not meant to be physically realistic)

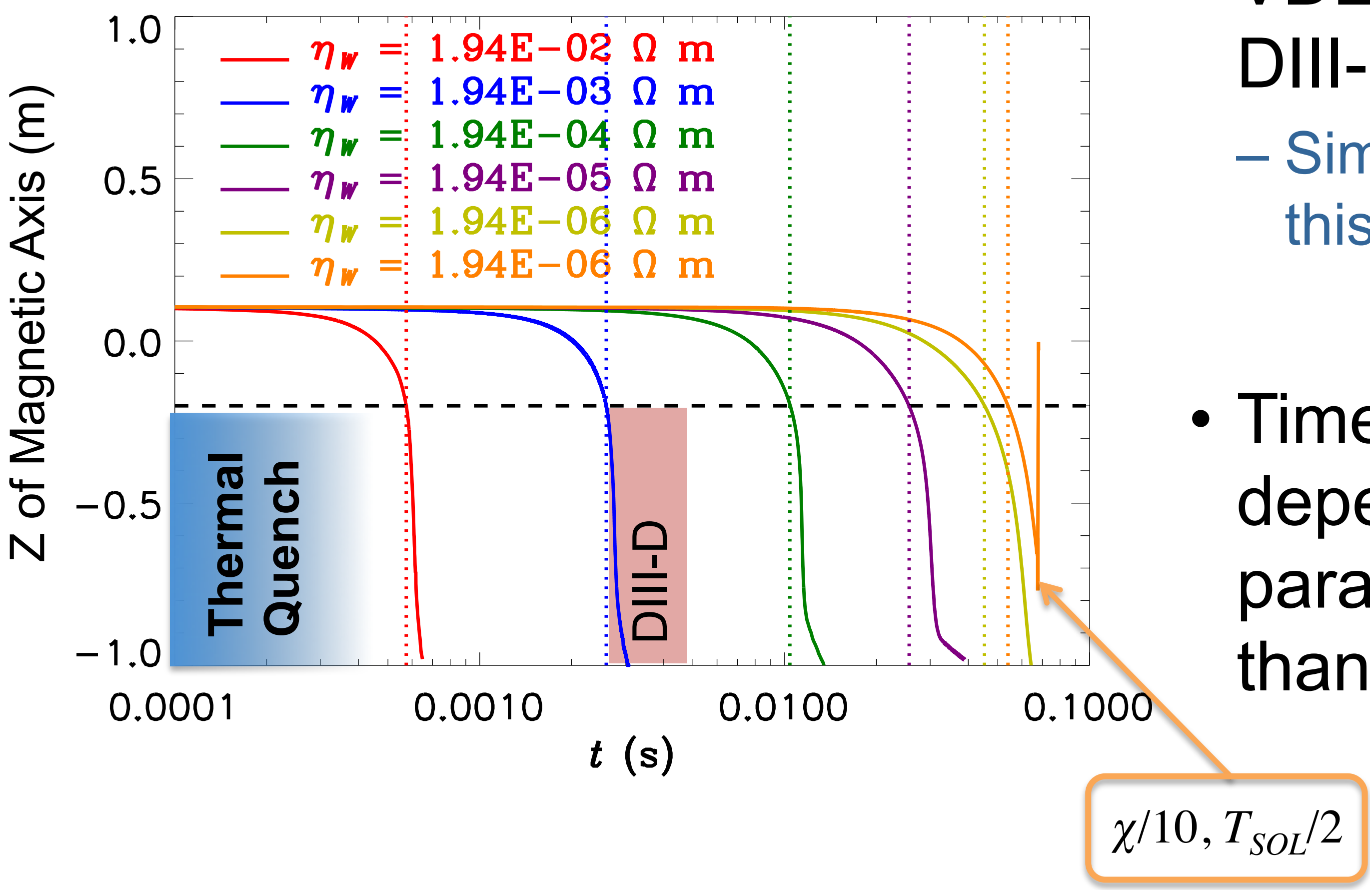


Strong Currents Form in Halo Region; Response Currents form in Wall and SOL

- Both co-I_P and counter-I_P currents are seen in the open field-line region



Timescale of VDE Is Determined by Wall Resistivity (η_W)



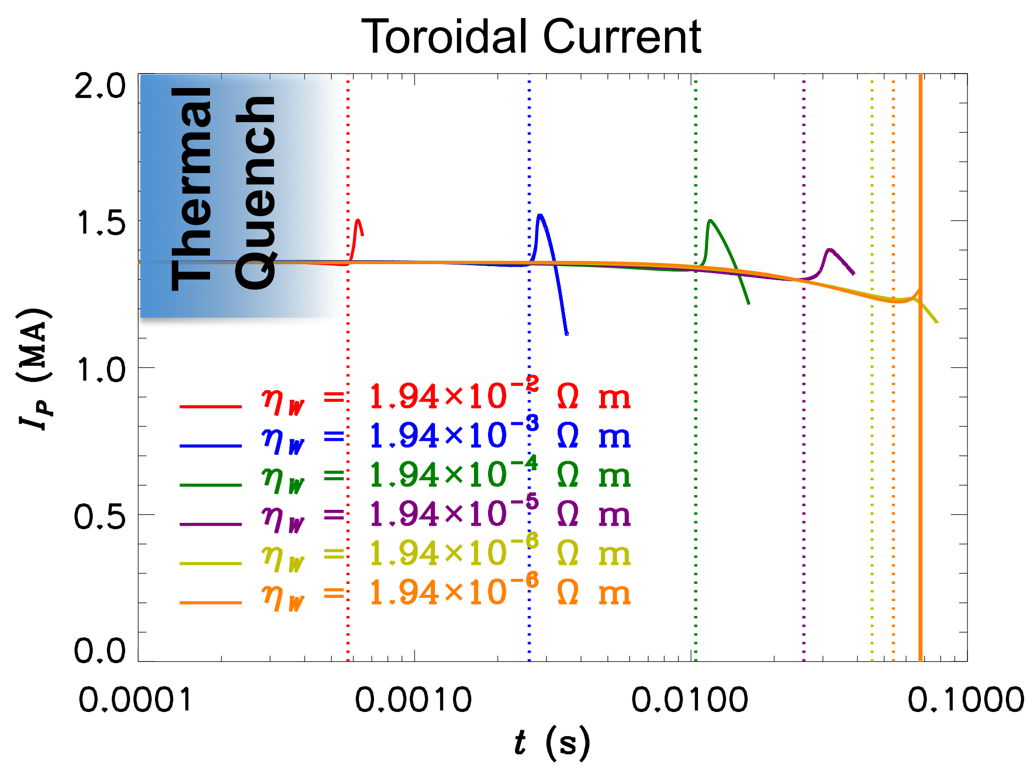
- Physically realistic VDE timescale in DIII-D is a few ms
  - Simulations bracket this regime
- Timescale weakly dependent on parameters other than η_W

Current Spike Observed Just Before Current Quench; Related to Vertical Motion of Plasma

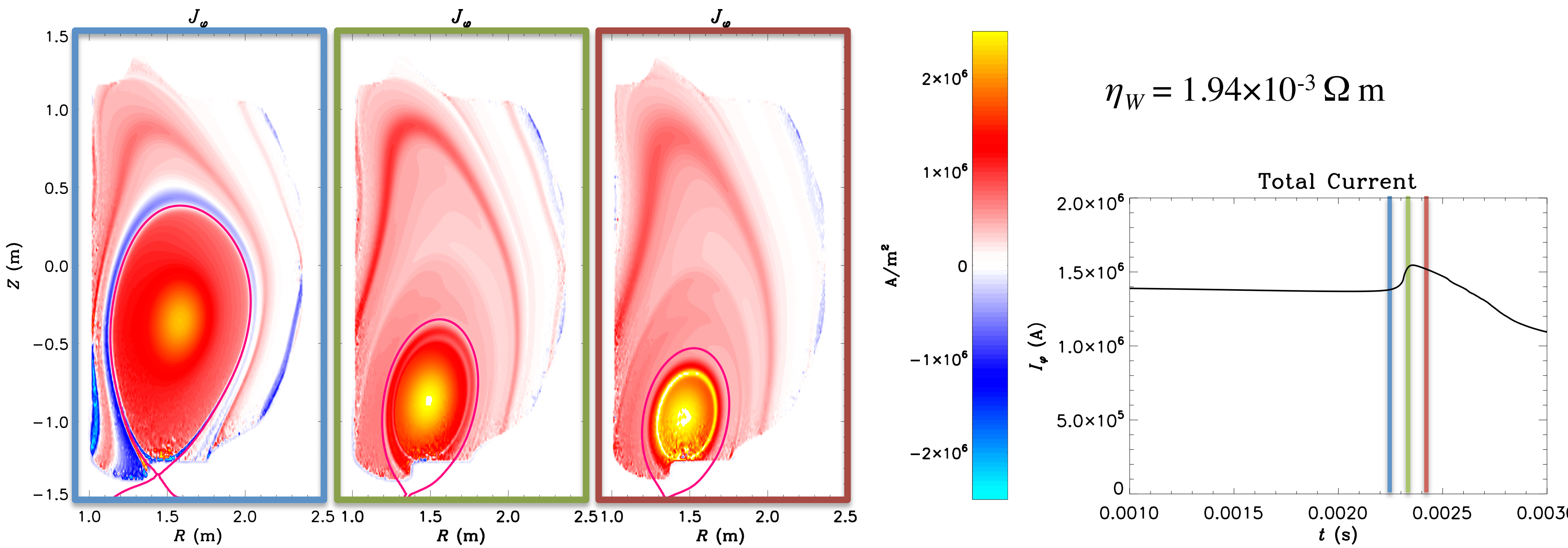
- Current spike occurs soon after plasma makes contact with the wall

- There is no spike associated with the thermal quench

- Spike is smaller when $\eta_W < \eta_{SOL}$



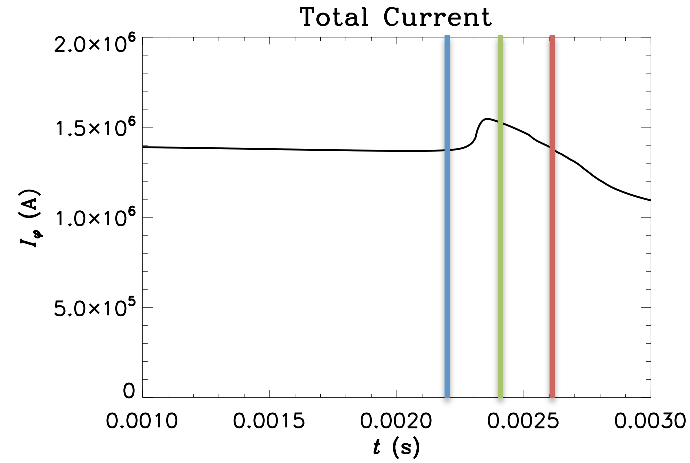
Current Spike Results from Loss of Induced Counter-I_P Currents When Plasma Contacts Wall



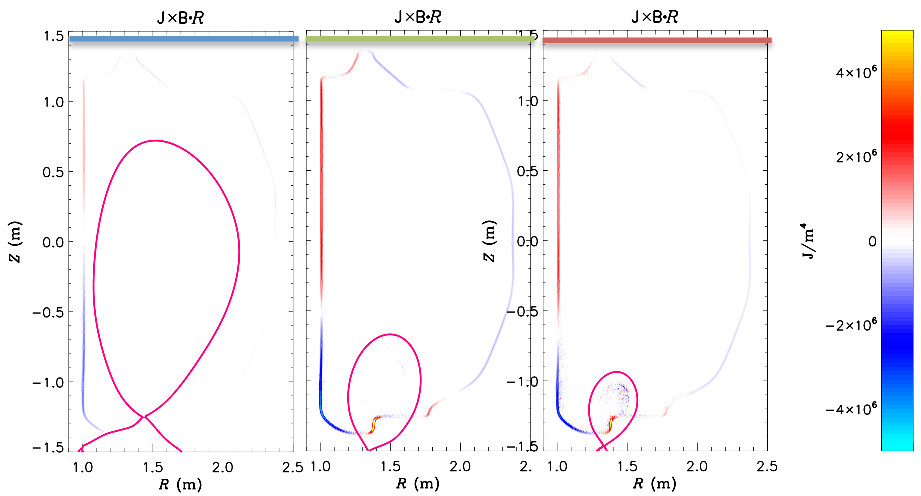
- Counter-I_P response currents are induced by motion of leading edge of plasma
- When plasma contacts wall, these currents quickly dissipate
- Eventually (after spike), toroidal current in wall flips sign to oppose I_P decay

Axisymmetric Forces Reach Maximum Just After Current Spike

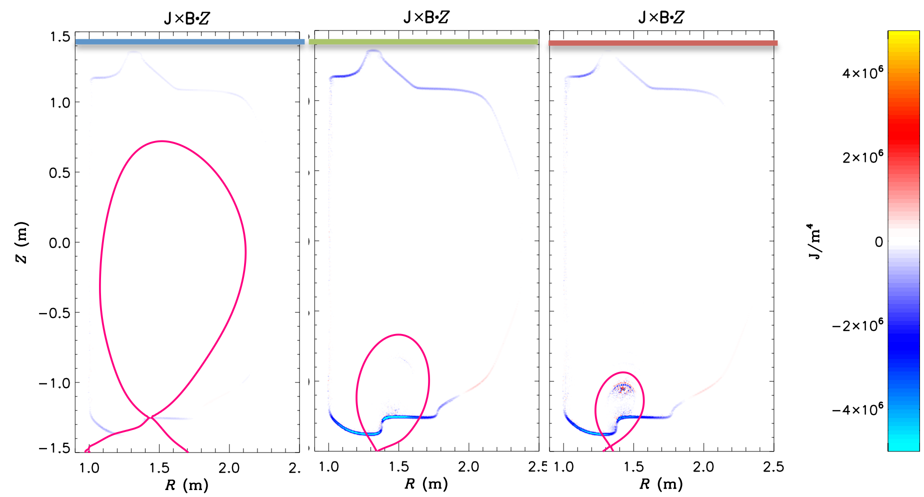
- Axisymmetric forces peak at ~100 kN /m^2
- Force distribution does not evolve significantly
- Currents in plasma are strong, but mostly force-free



Radial J×B Force



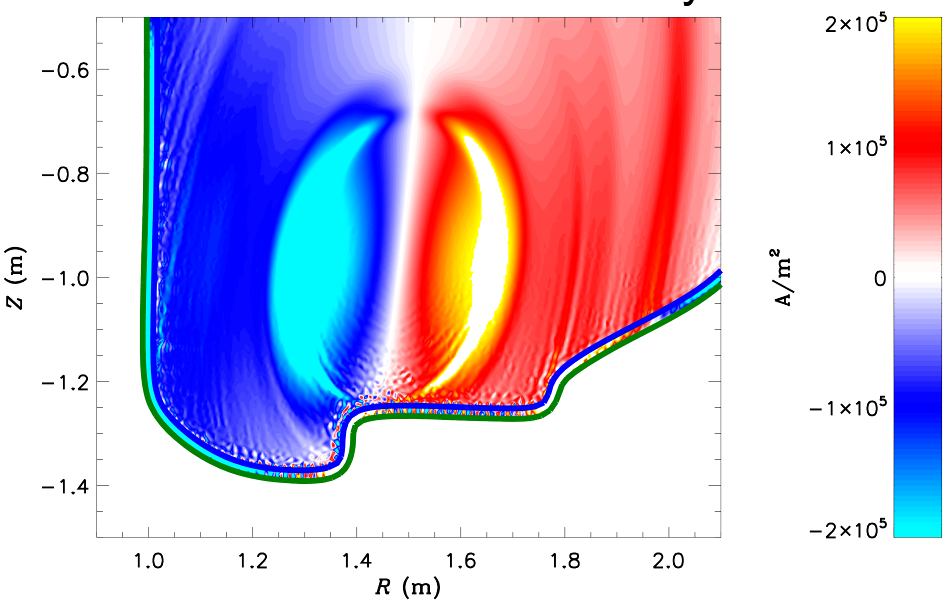
Vertical J×B Force



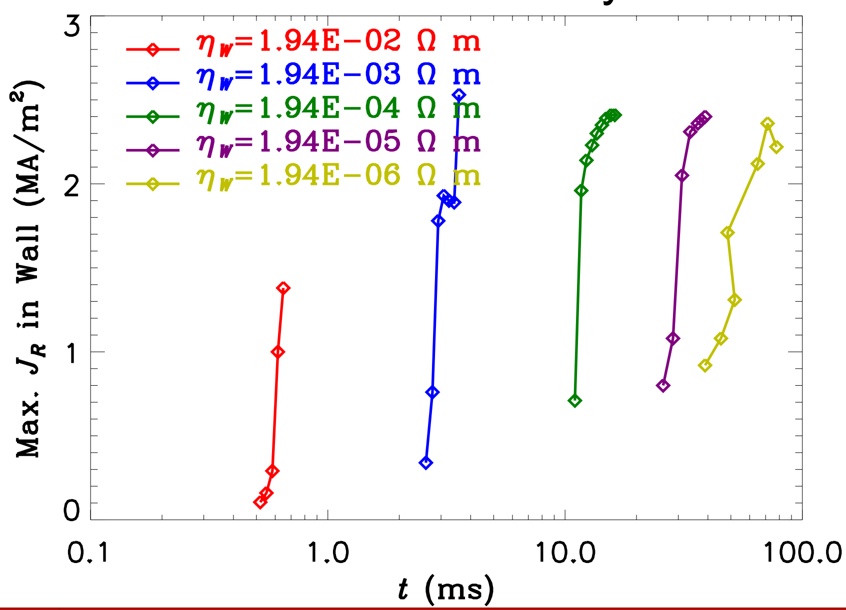
Maximum Axisymmetric Halo Currents and Wall Force Depend Weakly on η_W

- Halo currents can exceed 100 kA/m^2; observed both on divertor floor and center post
  - Distribution likely depends on temperature (resistivity) of open field-line region
- Maximum Halo currents and force density in the wall is only weakly dependent on wall resistivity
- Impulse to vessel increases with τ_W because force is applied for longer time

Vertical Current Density



Radial Current Density in Wall



3D Evolution Depends on Thermal History of Plasma

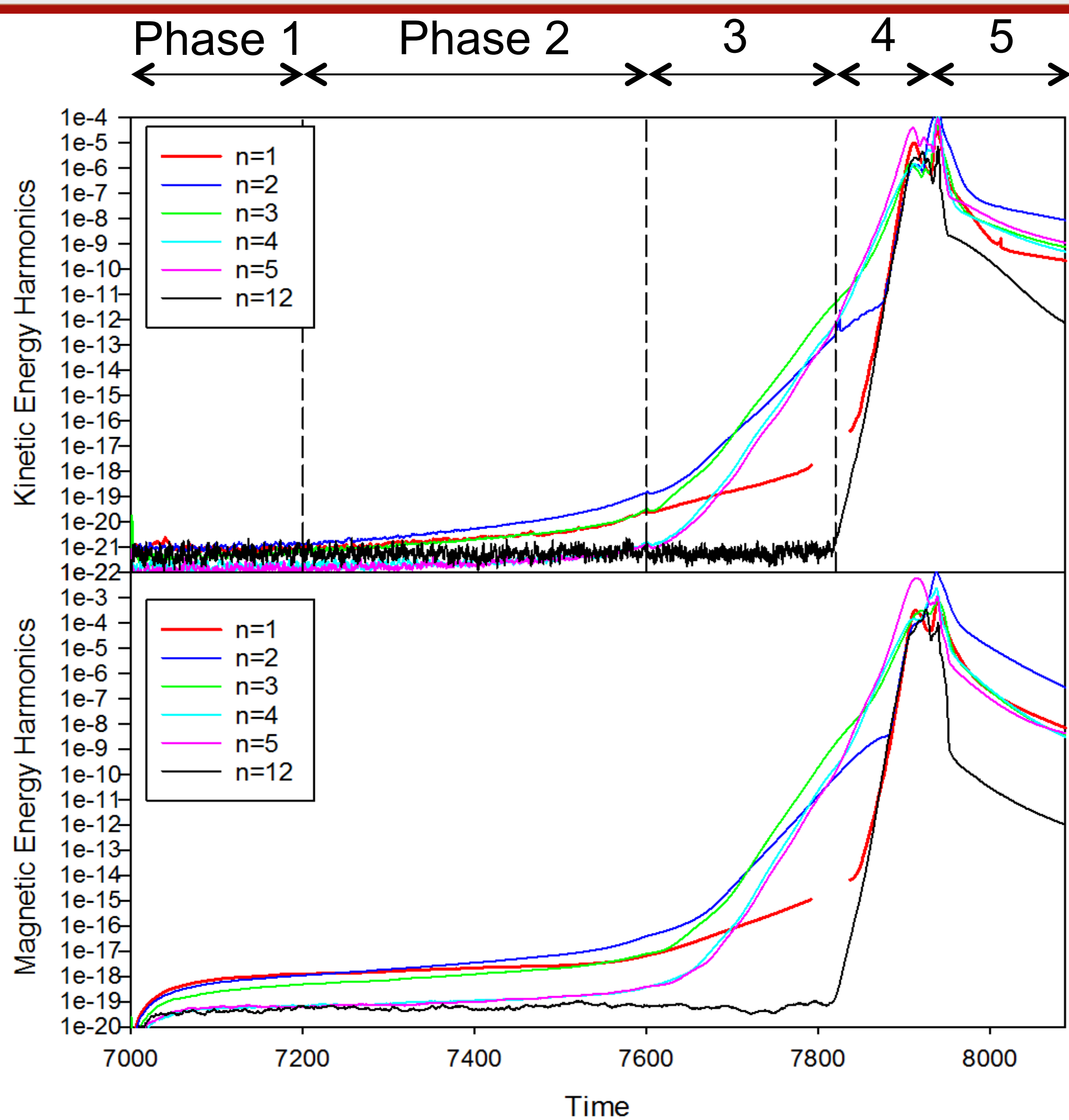
- Two competing effects determine q_{edge} once plasma is limited:
  1. q_{edge} drops as plasma shrinks and is scraped off by limiter
  2. q_{edge} rises because of resistive decay of I_P

- In cold-VDE (TQ happens before VDE), resistive decay is fast and q_{edge} rises
  - Plasma remains stable to $n > 0$ MHD

- In hot-VDE (no TQ before VDE), resistive decay is slow and q_{edge} drops
  - Plasma eventually becomes unstable to $n > 0$ MHD
  - $n > 0$ instability potentially causes strong Halo currents, wall forces, and TQ

- 3D simulations are expedited by testing linear stability of 2D simulations; then turning on 3D model when instability is found

3D Nonlinear Hot-VDE Calculation Shows Development and Saturation of 3D Modes



Phase 1:
Axisymmetric

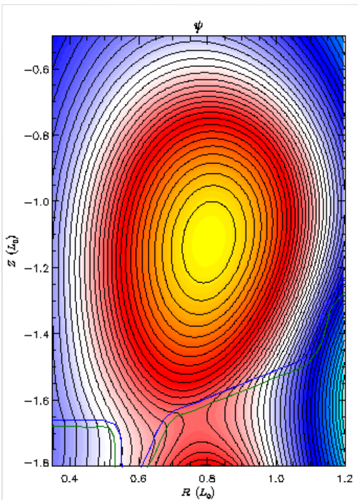
Phase 2:
n=2 tearing? mode dominates

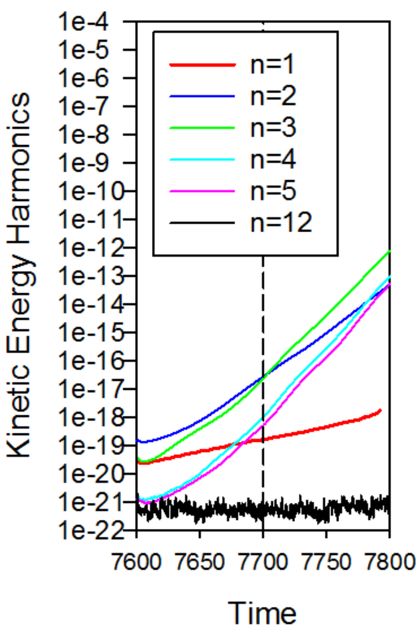
Phase 3:
n=3 tearing? mode begins to dominate

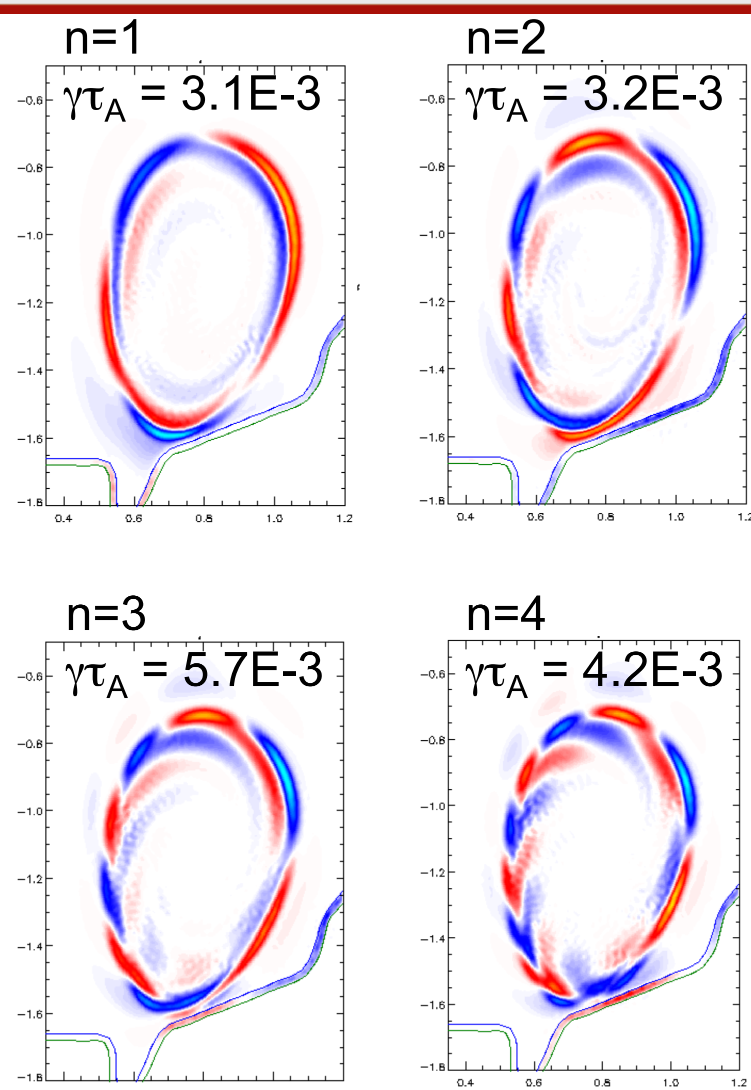
Phase 4:
n=1 and higher-n modes begin to grow

Phase 5:
Plasma gets scraped off and strongly wall stabilized

Linear Stability Analysis Finds Agreement With Nonlinear Calculation







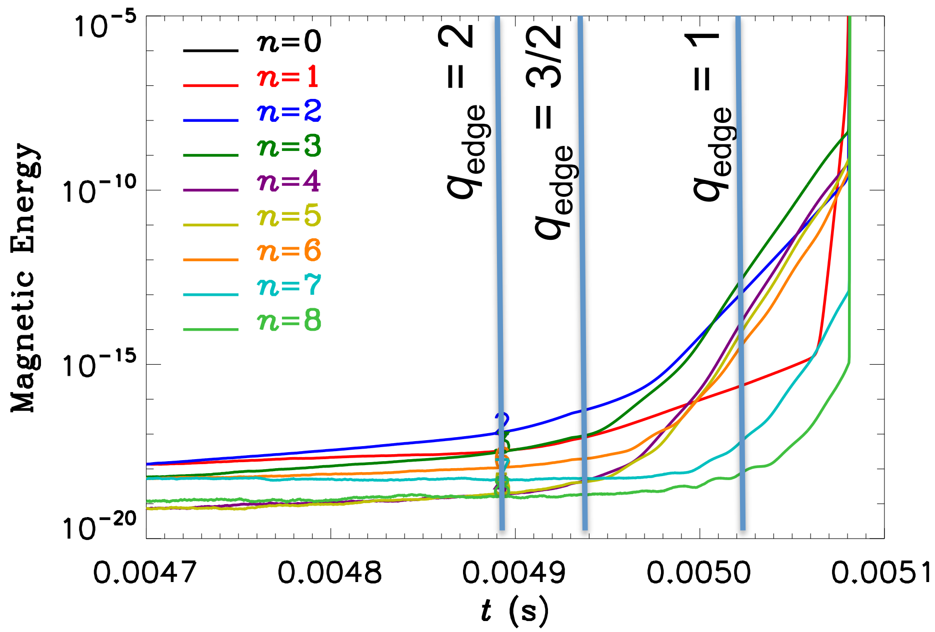
n≥5 stable

- Linear stability of axisymmetric solution is calculated at $t = 7700\ \tau_A$
  – Evolution of q profile in 2D and 3D cases is nearly identical
- Linear stability finds unstable low-n modes before nonlinear calculation does
- Growth rates are relatively small

In Hot-VDE Simulations, $q_{\text{edge}} < 1$ Before Non-Axisymmetry Becomes Significant

- Non-axisymmetric modes start growing when $q_{\text{edge}}=2$, but are still at small amplitude when $q_{\text{edge}}=1$

- For these cases, non-axisymmetric wall forces are small and highly localized near divertor
  - This is good news for ITER
  - Might not be the case for disruptions caused by non-axisymmetric instabilities
  - Might not be the case when non-axisymmetry of conducting structures is considered



Summary

- Nonlinear models of VDEs provide quantitative estimates of wall forces and halo currents
  - Preliminary comparisons with NSTX data show excellent agreement with halo current magnitude
  - Non-axisymmetric forces from VDE are small and localized (in these cases)

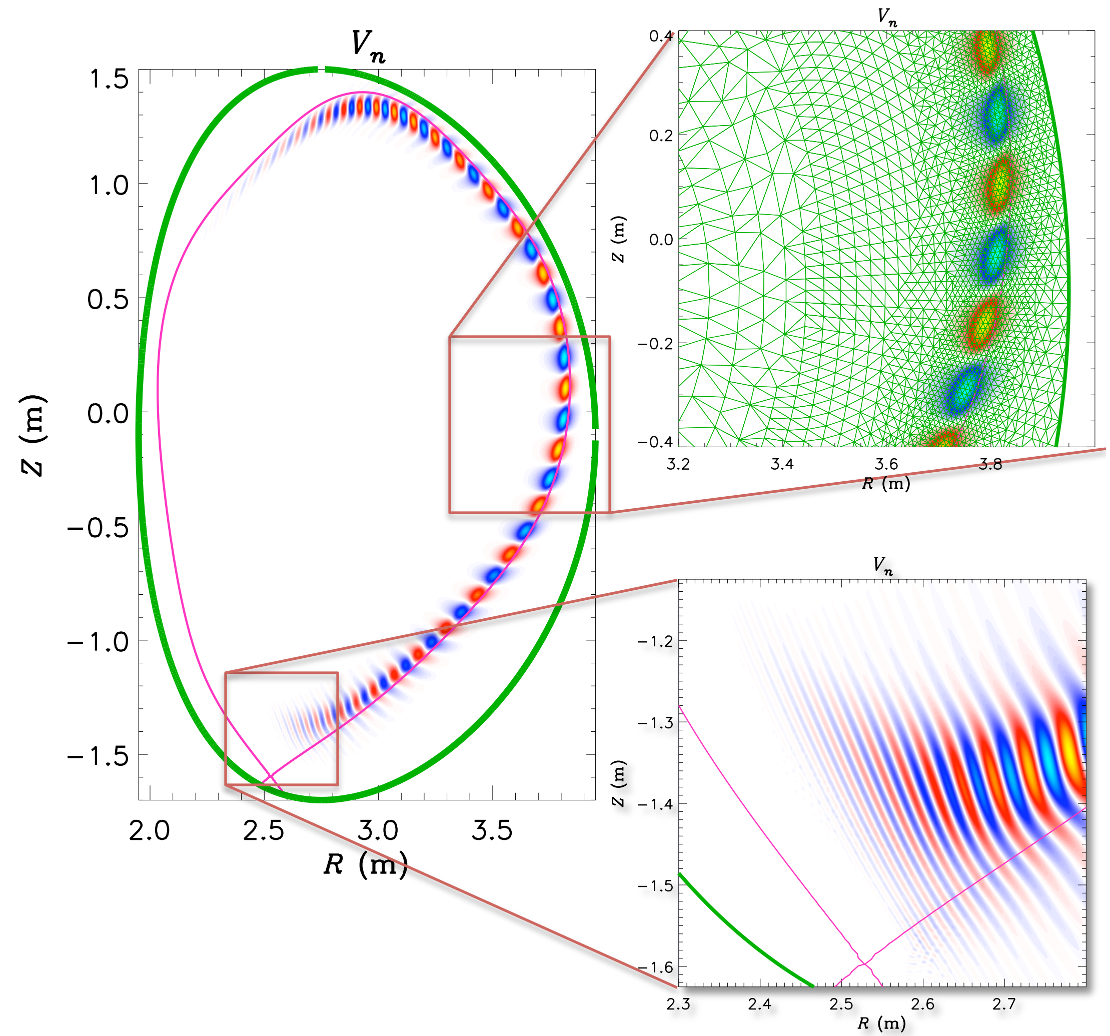
- Thermal history influences non-axisymmetric evolution of VDE
  - If plasma is cold before CQ, plasma remains kink-stable

- Still lots of unanswered questions
  - How do we know how close we are to a disruptive instability threshold?
    - Many linear stability thresholds can be crossed without disrupting
  - How can we mitigate the effects of disruptions?
  - How does disruption proceed when caused by a non-axisymmetric instability (like a locked mode)?

Extra Slides

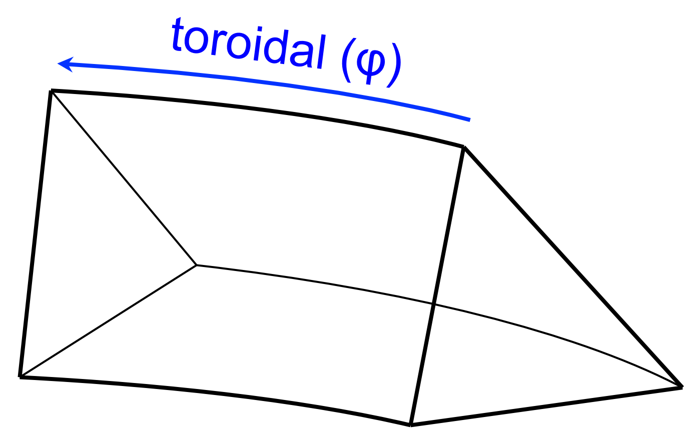
Mesh Packing Allows Fine Resolution In Regions of Interest

- M3D-C1 uses meshing software from SCOREC group at RPI
- Mesh can be packed anisotropically
- Triangular unstructured mesh allows field-aligned mesh packing with no problems near axis or x-point
- Mesh can be adapted dynamically (though we never do this)



High-Order C^1 Finite Elements

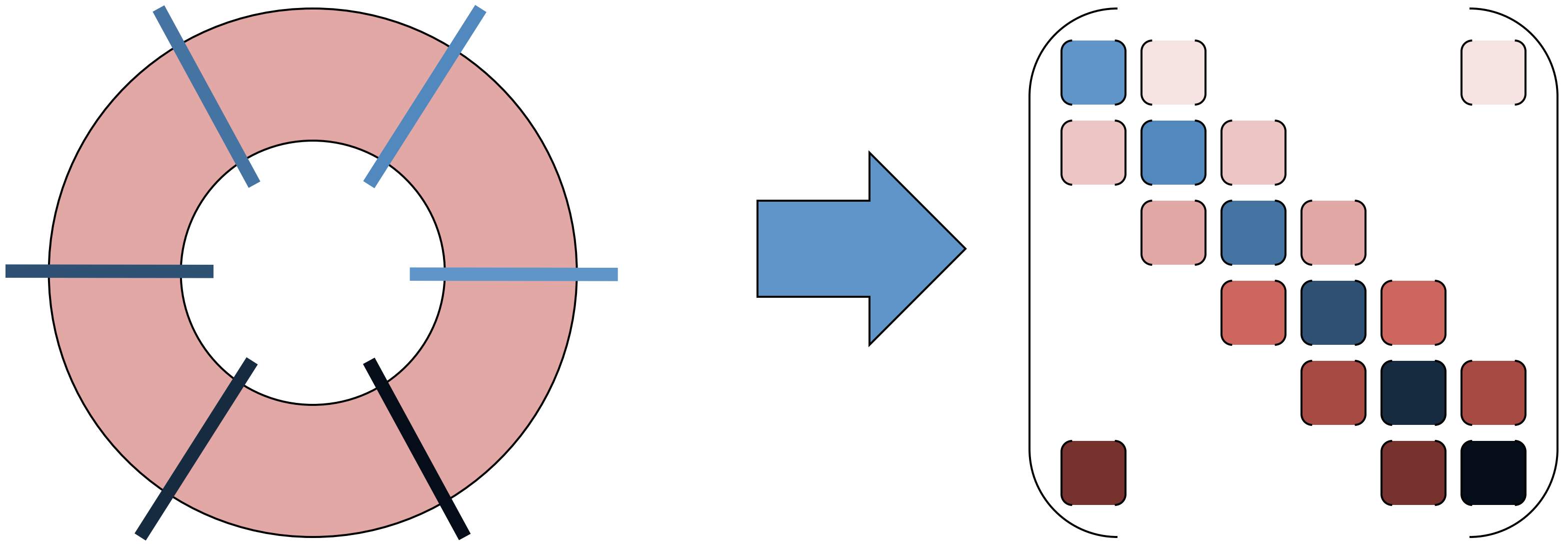
- Elements are a tensor product
  - Poloidally: 2D (triangular) reduced quintic elements
  - Toroidally: 1D cubic Hermite elements



- High-order elements lead to more compact matrices

- C^1 in all directions
  - Allows 4^{th} degree weak derivatives
  - Allows efficient use of flux/potential representation

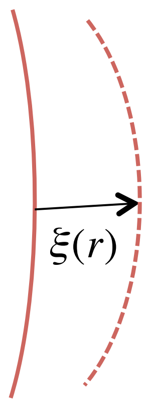
Hermite Elements in Toroidal Direction Yields Block Cyclic Tridiagonal Matrix



- Each plane yields a diagonal block
  - Only neighboring planes are coupled
  - Coupling is much stronger within planes than among planes (block diagonal dominant)
- Block-Jacobi preconditioning is effective
  - Diagonal block are factorized directly using SuperLU or MUMPS
  - This method is now available in PETSc. Thanks H. Zhang!

Assumption of Linearity May Be Suspect Near Edge and Rational Surfaces

- "Displacement" may be defined by movement of isotherms:



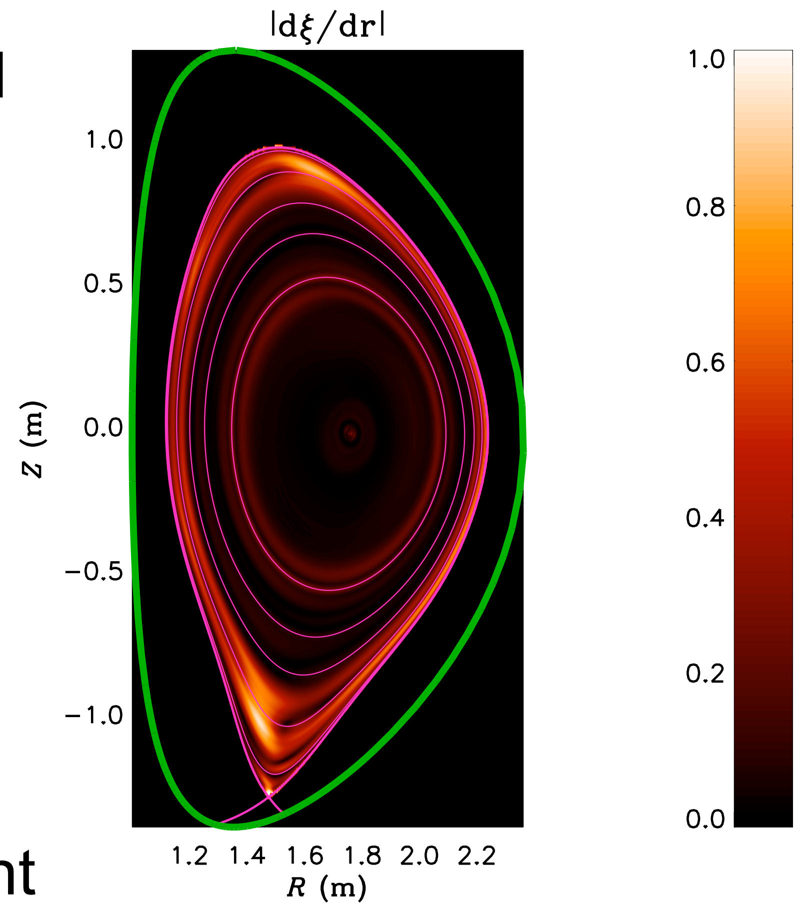
$$T_0(r+\xi) + \delta T(r+\xi) = T_0(r)$$

$$\left[T_0(r) + \frac{dT_0}{dr}\xi\right] + \delta T(r) = T_0(r)$$

$$\xi = -\frac{\delta T}{dT_0/dr}$$

- Overlap of adjacent surfaces is possible, especially near mode-rational surfaces, edge, & x-point

Overlap criterion: $\left|\frac{d\xi}{dr}\right| > 1$



DIII-D 117327 *n*=1